SOME ENERGY CHARACTERISTICS OF THE INDUCED INTERACTION OF MOVING CONDUCTING WALLS WITH EXTERNAL ELECTRIC WINDINGS FOR FINITE R_m

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In this article we consider a typical problem of the induced interaction for moving clusters in a conducting medium with external electric circuits.

The conducting clusters in this model are two plane-parallel solid walls having a finite thickness and a constant conductivity; these walls undergo specified oscillatory motion which is symmetric with respect to the median plane. Currents in the external electric circuit create a uniform magnetic field; the magitude of the currents is determined by conditions in electric circuits themselves and by the induced interaction of the currents in the conducting medium.

Using integral transformations, the equations for magnetic-field diffusion within the conducting wall and the Kirchhoff equations for the external electric circuits with corresponding additional conditions reduce to a system of ordinary differential equations which are solved numerically. The energy characteristics of the interaction process in the periodic mode of system operation are calculated (the amount of work done by the moving wall against the electric volume forces, and the amount of Joule losses in the conducting wall). It is shown that by exciting a magnetic field in the system with the aid of external emf sources it is possible to considerably reduce the Joule losses in comparison with the case of the external electric circuit closed by an ohmic load.

1. An electric current

$$= \sigma\left(\mathbf{E} + \frac{\mathbf{V} \times \mathbf{H}}{c}\right) \tag{1.1}$$

produced by the motion of a conducting medium in an electromagnetic field gives rise to an electric body force (EBF) $\mathbf{f} = c^{-1}[\mathbf{j} \times \mathbf{H}]$ and Joule heat.

The work done by the moving medium against the EBF, ignoring Joule losses in the conducting medium itself (i.e., useful work), can be extracted from the system in the form of electric energy; therefore, this quantity is an important energy chanracteristic of the interaction process. In the local sense (when the mechanical work $A = c^{-1}[\mathbf{j} \times \mathbf{H}] \cdot \mathbf{V}$ and the Joule heat $Q = j^2/\sigma$ refer to a unit of volume of medium and a unit of time) the difference $A - Q$ is determined to a considerable degree by the relationship between the vectors **E** and $\mathbf{E}$ и $c^{-1}[\mathbf{V} \times \mathbf{H}]$ (Eq. (1.1); when the three vectors **H**, **E** and **V** are mutually perpendicular, we have [1]

$$P = A - Q = kA, \quad A = (1-k)c^{-2}\,\sigma V^2 H^2, \tag{1.2}$$

where the parameter k characterizes the magnitude and direction of **E** in comparison with $c^{-1}(\mathbf{V} \times \mathbf{H})$, and is given by the relation $\mathbf{E} = -kc^{-1}(\mathbf{V} \times \mathbf{H})$.

It is now clear that for positive work A, the useful P can either be positive or negative (Q greater than A); this is entirely dependent on the relationship between the two terms in (1.1).

It is clear that any device intended for converting the energy of a moving conducting medium into electric energy must posses the ability to regulate **E** in comparison with $c^{-1}(\mathbf{V} \times \mathbf{H})$. In the case of a conduction MHF generator, this is achieved by changing the external load; as a result, the voltage between electrodes changes; this causes a change in the electric field in the MHD channel. If we consider the induced interaction between moving clusters of a conducting medium (or a medium with a periodic conductivity distribution) and a magnetic field, we can only calculate the eddy electric field resulting from the change in time of magnetic field **H**.

Here, if the external magnetic field is time-constant, it can easily be shown from the Maxwell equations that when $R_m \ll 1$ the magnetic-field perturbation owing to currents in the clusters is small and the magnitude of the electric-field strength is $E \sim R_m c^{-1} VH$, i. e., $|k| \sim R_m \ll 1$; this is independent of the parameters of the outer loop through which energy is drawn. Here, as is clear from (1.2), almost all work done against the EBF is dissipated in the form of Joule heat; this explains the low efficiency of the pulsed-conduction MHD generators considered in [2, 3] in comparison with conduction generators. It is now clear that in order to increase the efficiency of similar devices it is necessaty to use a variable external magnetic field (external winding is closed either through a variable emf or is part of the tank circuit). Then, by changing the amplitude of H and the phase shift between cluster oscillations and the external emf we can control the parameter k and, consequently, increase the energy conversion efficiency.

The discussion which follows shows that it is also possible to improve the energy characteristics for induced interaction of clusters with electric windings by using an external emf for finite R_m when, in principle, k can be regulated by the parameters of the external load.

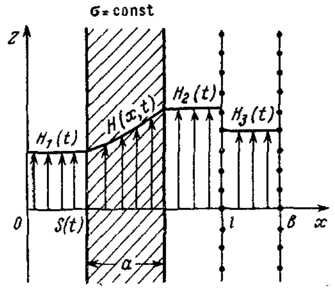

Fig. 1

2. Statement of the problem. Equations. Figure 1 schematically shows the model under consideration in which two plane solid walls of finite thickness a serve as the moving clusters; each of two electric windings constitutes a pair planes $x = \pm$ const with loops parallel to the y-axis (because the system is symmetric with respect to the $x = 0$ plane, Fig. 1 shows the half-plane $x > 0$). Notation for the geometric quantities is also given; it is necessary to remember that the motion of the second wall is absolutely symmetric to the first one with respect to the plane of symmetry $x = 0$. The one-dimensional approximation will be used; therefore, it is assumed that the wall dimensions along the y and z-axes are large and the loops $x = \pm b$ is short-circuited (i. e., the ohmic resistance is zero) and the internal winding $x = \pm \Delta l$ is either closed through an ohmic resistance or an external sinusoidal emf is couplied to it.

Wall motion is specified. The law of motion governing the wall is depicted in the form of a graph in Fig. 2, from which it is clear that the wall undergoes periodic oscillatory motion with constant velocity between the extreme positions S_0 and S_1 (of course, $S_1 + a < l$).

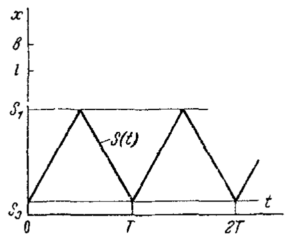

Fig. 2

To be specific, we let $S(0) = S_{min} = S_0$ for $t = 0$ and assume that the magnetic-field distribution is uniform in the regions $-l < x < l$ and $l < x < l$. Depending on the magnitude of the initial currents in the winding there may be a discontinuity at the inner winding $x = \pm l$.

The probelm consists of finding the magnetic-field strength $\mathbf{H} = [0, 0; H(x, t)]$, the current density in the conducting wall $\mathbf{j} = [0; j(x, t); 0]$, and the current in the windings for $t > 0$; here, from symmetry considerations, $H(-x, t) = \mathbf{H}(x, t)$, $j(-x, t) = -j(x, t)$. Calculations show that the system becomes periodic after conducting walls undergo several oscillation cycles. The energy characteristics of the interaction process in the period mode are of particular interest; the overall values for the mechanical work done against the EVF and the Joule losses over the entire thickness of one wall were calculated for one oscillation cycle and referred to a unit of area in the yz-plane. Below these quantities will be denoted by A_* and Q_*.

We shall make use of the following assumptions: (a) the wall conductivity σ is constant; (b) the effect of bias currents in the nonconducting regions $0 < x < s(t)$ and $S + a < x < b$ on the magnitude of the magnetic field in the system is negligible. This is true for "technical" velocities $V \ll c$. A similar problem allowing for bias currents has been considered in [4] for the case of an infinite wall conductivity and without induced interaction with the electric windings; in this article, the motion of a conducting layer was studied allowing for radiation.

The basic equations are:

$$\begin{gathered}
\frac{\partial H(x_1, t)}{\partial t} = \frac{c^2}{4\pi\sigma}\frac{\partial^2 H(x_1, t)}{\partial x_1^2} \qquad (0 < x_1 < a); \\
\frac{c^2}{4\pi\sigma}\frac{\partial H(x_1, t)}{\partial x_1}\bigg|_{x_1=0} = \frac{d}{dt}[H_1(t)S(t)]; \\
\mathscr{E}(t) - \frac{2n}{c}\frac{c^2}{4\pi\sigma}\frac{\partial H(x_1, t)}{\partial x_1}\bigg|_{x_1=a} - \frac{2n}{c}\frac{d}{dt}[(l - a - S)H_2(t)] = \mathcal{J}R; \\
(b - l)\frac{d}{dt}\left[H_3(t) - \frac{4\pi\eta}{c}\mathcal{J}(t)\right] - \frac{Rc}{2n}\mathcal{J}(t) + \frac{c}{2n}\mathscr{E}(t) = 0.
\end{gathered} \tag{2.1}$$

Here $H(x_1, t)$ is the magnetic field strength in the wall $0 < x < S(t)$; $H_1(t)$ and $H_2(t)$ are the field strengths in, respectively, the nonconducting regions $0 < x < S(t)$ and $S(t) + a < x < l$ (in view of assumption (b) the field strength does not depend on the space coordinates); R and $\mathscr{E}(t)$ are the ohmic resistance and external emf in the winding $x = \pm 1$, referred to a unit of height along the z-axis (to n loops) and a unit of width along the y-axis (i.e., R and $\mathscr{E}(t)$ are referred to a unit of area in the yz-plane); and $\mathcal{J}(t)$ is the current force in the loop of the $\pm l$ winding.

Equation (2.1.1) is the induction equation in the variables x_1, t where $x_1 = x - S(t)$ is the system coordinate associated with the moving wall.

Equation (2.12) is the continuity equation for E_y at the boundary of $x = S(t)$ (in the coordinate system associated with the moving wall) [5]. Equation (2.1.3) is obtained from the equation

$$\mathscr{E}(t) - \frac{n}{c}\frac{d\Phi}{dt} = \mathcal{J}R \qquad \left(\Phi = 2\left[H_1(t)S(t) + \int_0^a H(x_1, t)\,dx_1 + (l - a - S)H_2(t)\right]\right),$$

where Φ is the magnetic flux through one loop of area $1.2l$, and

$$\frac{d}{dt}\left[H_1(t)S(t) + \int_0^a H(x_1, t)dx_1\right] = \frac{c^2}{4\pi\sigma}\frac{\partial H(x_1, t)}{\partial x_1}\bigg|_{x_1=a},$$

which is derived similarly to Eq. (2.1.2). Finally, Eq. (2.1.4) is the condition that the magnetic flux be constant:

$$H_1(t)S(t) + \int_0^a H(x_1, t)\,dx_1 + (l - a - S)H_2(t) + (b - l)H_3(t) = \text{const};$$

this equation is written using the relationship

$$H_3(t) - H_2(t) = -c^{-1}4\pi n\mathcal{J}(t).$$

In dimensionless form, system (2.1) becomes

$$\begin{gathered}
\frac{\partial h(\xi, \tau)}{\partial \tau} = \beta\frac{\partial^2 h(\xi, \tau)}{\partial \xi^2} \qquad (0 < \xi < 1), \\
\frac{d}{d\tau}[h_1(\tau)s(\tau)] = \beta\frac{\partial h(\xi, \tau)}{\partial \xi}\bigg|_{\xi=0},
\end{gathered} \tag{2.2}$$

$$\frac{d}{d\tau}[(l_1-1-s)\,h_2(\tau)]=\pi\delta\mathscr{E}_1(\tau)-\beta\left.\frac{\partial h(\xi,\tau)}{\partial\xi}\right|_{\xi=1}-\lambda i(\tau)\,,$$

$$\frac{d}{d\tau}[h_2(\tau)-i(\tau)]-\frac{\lambda}{b_1-l_1}i(\tau)+\frac{\pi\delta}{b_1-l_1}\mathscr{E}_1(\tau)=0\,,$$

$$\left(\tau=\frac{t}{T},\quad \xi=\frac{x_1}{a},\quad s=\frac{S}{a},\quad b_1=\frac{b}{a},\quad l_1=\frac{l}{a},\quad h=\frac{H}{H_0},\quad h_1=\frac{H_1}{H_0}\,,\right.$$

$$h_2=\frac{H_2}{H_0},\quad h_3=\frac{H_3}{H_0},\quad i=\frac{4\pi n\mathscr{I}}{cH_0},\quad \mathscr{E}_1=\frac{\mathscr{E}}{\mathscr{E}_0},\quad \beta=\frac{c^2T}{4\pi\sigma a^2},\quad \lambda=\frac{c^2RT}{8\pi n^2 a}\,,$$

$$\left.\delta=\frac{cT\mathscr{E}_0}{2\pi naH_0}\right)$$

Here T is the oscillation period, H_0 is the initial magnetic field strength is space $|x| < l$ and $\mathscr{E}_0$ is the amplitude of E(t).

For unknown $h(\xi,\tau)$, $h_1(\tau)$, $h_2(\tau)$ and $i(\tau)$, initial conditions are

$$h(\xi,0)=1,\quad h_1(0)=1,\quad h_2(0)=1,\quad i(0)=1-h_3(0)\,, \tag{2.3}$$

and the boundary conditions for $h(\xi,t)$ are

$$h(0,\tau)=h_1(\tau),\qquad h(1,\tau)=h_2(\tau)\,. \tag{2.4}$$

In order to obtain an approximate numerical solution to the problem (2.2) is reduced to a system of ordinary differential equations. To do this, the gradients $dh/d\xi\,|_{\xi=0}$ and $\partial h/\partial\xi\,|_{\xi=1}$, in (2.2) are expressed in terms of $h_1(\tau)$ using Eq. (2.2.1), conditions (2.4), and initial condition $h(\xi,0)=1$. For this purpose, an integral transformation in the variable ξ [6] was employed with respect to Eq. (2.2.1); as a result, we obtained the solution

$$h(\xi,\tau)=h_1(\tau)+[h_2(\tau)-h_1(\tau)]\,\xi+\sum_{\gamma=1}^{\infty}U_\gamma(\tau)\sin\gamma\pi\xi\,, \tag{2.5}$$

where $U_\gamma(\tau)$ is the "image" of the function $u(\xi,\tau)=h(\xi,\tau)-[h_2(\tau)-h_1(\tau)]\xi$, which is given by the differential equation

$$\frac{dU_\gamma(\tau)}{d\tau}+\beta(\gamma\pi)^2U_\gamma(\tau)=-\frac{2}{\gamma\pi}\frac{d}{d\tau}[h_1(\tau)-(-1)^\gamma h_2(\tau)]\qquad(\gamma=1,2,\ldots) \tag{2.6}$$

and the zero initial condition $U_\gamma(0)=0$. The gradients $\partial h/\partial\xi\,|_{\xi=0}$ and $\partial h/\partial\xi\,|_{\xi=1}$ are found by term-by-term differentiation of (2.5). Equation (2.2.2)–(2.2.4), together with Eqs. (2.6) and (2.5), make up the sought system of ordinary differential equations.

On the basis of Fig. 2, the function $s(\tau)$ in the equation is given by

$$s(\tau)=s_0+2(s_1-s_0)(\tau-n)\qquad\text{for } n<\tau<n+{}^1\!/_2$$
$$s(\tau)=s_1-2(s_1-s_0)[\tau-(n+{}^1\!/_2)]\qquad\text{for } n+{}^1\!/_2<\tau<n+1\,,$$

where n = 0, 1, 2, ... are natural numbers corresponding to the cycle sequence and the external emf has the same period as the oscillation period T; $\mathscr{E}_1(\tau)=\sin(2\pi\tau+\theta)$, where θ is the phase shift between the applied emf and the oscillations of the conducting walls. From the energy point of view, the case of short periods is much less suitable, as shown by calculations.

3. Numerical results and conclusions. The system of equations obtained was solved numerically using the Runge-Kutta method; in this case, the number of terms in the series in (2.5) varied from 15 to 30.

The sought functions, with a specified accuracy, become periodic after several cycles of wall oscillations; here the greater the dimensionless parameter β, the smaller the required number of cycles. It is necessary to note that the parameter β is directly associated with the number R_m:

$$R_m=\frac{4\pi\sigma Va}{c^2}=\frac{4\pi\sigma a^2}{c^2T}\frac{VT}{a}=\frac{1}{\beta}\,2(s_1-s_0)\,.$$

We shall write formulas for calculating the energy quantities (referred to a unit of area in the yz-plane). Since all points of the conducting wall have the same velocity, the total work A_* is equal to the work done against the total force:

$$F_x = -\int_0^a \frac{\partial}{\partial x_1}\left(\frac{H^2}{8\pi}\right) dx_1 = -\frac{1}{8\pi}\left[H_2^2(t) - H_1^2(t)\right] = -\frac{H_0^2}{8\pi}\left[h_2^2(\tau) - h_1^2(\tau)\right],$$

i.e.,

$$A_* = -\int_t^{t+T} F_x \frac{dS}{dt}\, dt = \frac{H_0^2 a}{8\pi}\int_\tau^{\tau+1} \frac{ds}{d\tau}\left[h_2^2(\tau) - h_1^2(\tau)\right] d\tau\,, \tag{3.1}$$

Within one period, twice the value of the useful work $P_* = A_* - Q_*$ is equal to the electric energy liberated in an element of winding $x = \pm\, l$:

$$W_* = \int_t^{t+T}\left[\mathcal{J}^2 R - \mathscr{E}(t)\,\mathcal{J}(t)\right] dt = \frac{H_0^2 a}{2}\left[\frac{\lambda}{\pi}\int_\tau^{\tau+1} i^2(\tau)\, d\tau - \delta\int_\tau^{\tau+1} \mathscr{E}_1(\tau)\, i(\tau)\, d\tau\right] \tag{3.2}$$

($W_* = 2P_*$ because two conducting walls work on one electric winding); the electrical efficiency is $\eta = W_*/2A_*$.

Two cases will be considered.

(a) Winding $x = \pm\, l$ is closed by an external emf and the ohmic resistance is negligible, i.e., $\lambda \ll 1$ (λ is the reciprocal of the Q of this winding). Since $i(\tau) \sim h_2(\tau)$, when $\lambda \ll 1$ the last term $\lambda i(\tau)$ in (2.2.3) can be ignored in comparison with the term $(ds/d\tau)h_2(\tau)$. Then Eqs. (2.2.2), (2.2.3), (2.6), and (2.5) can be treated independently of (2.2.4) and $h_1(\tau)$, $h_2(\tau)$, and $U_\gamma(\tau)$ can be found whether or not there is an external winding $x = \pm\, b$. We then use Eq. (2.2.4) to find the current force $i(\tau)$ from known $h_2(\tau)$ and $\mathscr{E}_1(\tau)$:

$$i(\tau) = -h_3(0) + h_2(\tau) + \delta\frac{\pi}{b_1 - l_1}\int_0^\tau \mathscr{E}_1(\tau)\, d\tau\,. \tag{3.3}$$

Substituting this expression into (3.2) for $\lambda = 0$ we obtain

$$W_* = -\frac{H_0^2 a}{2}\,\delta\int_\tau^{\tau+1} \sin(2\pi\tau + \theta)\, h_2(\tau)\, d\tau\,,$$

since the remaining terms in (3.3) drop out upon integration.

Since when there is no winding $x = \pm\, b$ we have $i(\tau) = h_2(\tau)$, it is clear that for $\lambda = 0$ the presence of this winding does not affect the energy characteristics of the interaction process and can only serve to regulate the power coefficient which is defined as the ratio

$$\int_t^{t+T} \mathscr{E}(t)\mathcal{J}(t)\, dt \Bigg/ \sqrt{\int_t^{t+T} \mathscr{E}^2(t)\, dt \int_t^{t+T} \mathcal{J}^2(t)\, dt} =$$

$$= \sqrt{2}\int_\tau^{\tau+1} \sin(2\pi\tau + \theta)\, i(\tau)\, d\tau \Bigg/ \sqrt{\int_\tau^{\tau+1} i^2(\tau)\, d\tau}\,.$$

Numerical calculations were performed for one geometry with dimensionless parameters $s_0 = 0.1$, $s_1 = 2.5$, $l = 4$, and $b_1 = 5$. The dimensionless parameter δ characterizing the amplitude of the change in magnetic field (due to an applied emf) with respect to the field H_0 is taken equal to 1. The variables are the parameters $\beta(R_m)$ and θ. As an illustration of a system becoming periodic, Fig. 3 shows the curves $h_1(\tau)$ and $h_2(\tau)$ for $\beta = 0.5(R_m = 9.6)$, $\theta = 3\pi/4$, and $\lambda = 0$ from the very beginning. The subscripts 1, 2, ... corresponding to the oscillation cycles of the conducting layer. Thus, for example, curves with subscript 1 apply to the time interval from $\tau = 0$ to $\tau = 1$, curves 2 apply to the interval $1 \ll \tau \ll 2$, etc. All curves at point $\tau_* = n + 1/2$ undergo a discontinuity in the derivative, which is caused by a directional change in velocity. Curves obtained for large τ are not shown on the graph because it is hard to distinguish them from curve 4.

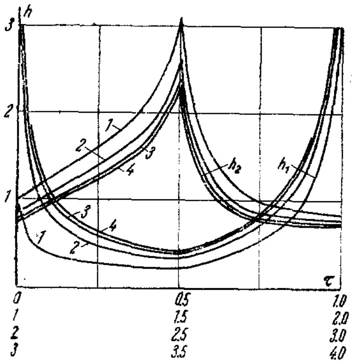

Fig. 3

Figures 4 and 5 show the curve $h_1(\tau)$ and $h_2(\tau)$ for different β and θ after the periodic mode has been reached.

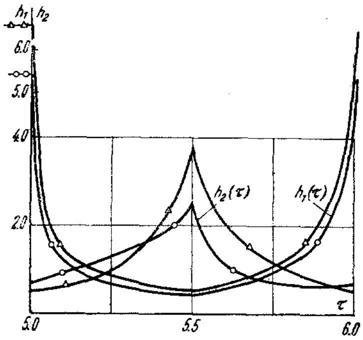

Fig. 4

The parameters in Fig. 4 are $\beta = 0.5(R_m = 9.6)$ $\lambda = 0$ (the circles and triangles apply respectively to $\theta = 5\pi/8$ and $\theta = 7\pi/4$) while the parameters in Fig. 5 are $\beta = 2(R_m = 2.4)$ $\lambda = 0$ (the circles, triangles, and crosses correspond respectively to $\theta = \lambda/4, \theta = \pi/4, \theta = \pi/2 + \pi/8$, $\theta = \pi 2.4$).

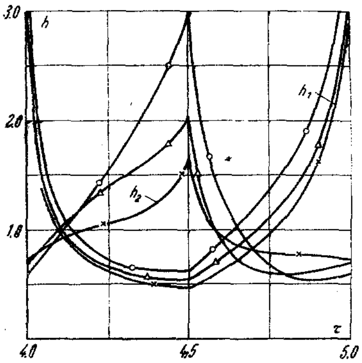

Fig. 5

For purposes of clarity, the graph of the current force i (τ) as a function of time, as calculated using (3.3) (in the periodic mode) for one value $\theta = 3\pi/4$ and for $\beta = 2(R_m\ 2.4)$ is shown in Fig. 6 (for this value of θ the quantity η is approximately a maximum for $\beta = 2$). It was assumed that the constant $h_3(0) = 1$ in (3.3), i.e., i $(0) = 0$. Here the curve $E_1(\tau) = \sin(2\pi\tau + \theta)$ for the applied external emf is indicated by the dashed line.

The energy characteristics $w_* = 2W_*/H^2{}_0\, a$ and η as functions of θ are shown in Figs. 7–9 (Fig. 7 for $\beta = 0.2$, Fig. 8 for $\beta = 0,5$, and Fig. 9 for $\beta = 2$). As is clear from Fig. 8, the amount of electrical energy liberated per period as a function of the phase shift in θ can be both positive and negative (in this case, energy is required from the network). Since for $W_* < 0$, we have $\eta = W_*/2A_* < 0$, the work done against the EBF upon motion of the layer is positive, i.e., the mode for θ for which $W_* < 0$ represents not the motive mode but the mode occurring when the Joule losses within the conducting medium exceed the amount of work performed; therefore, part of the external

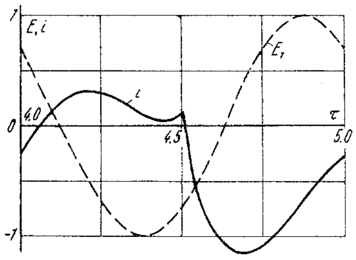

Fig. 6

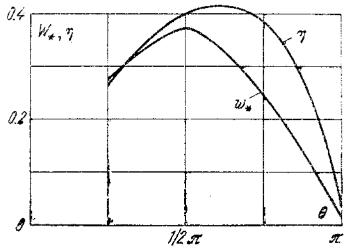

Fig 7

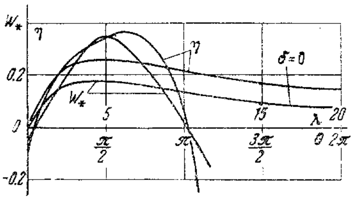

Fig 8

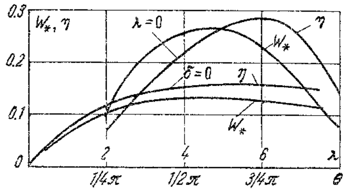

Fig 9

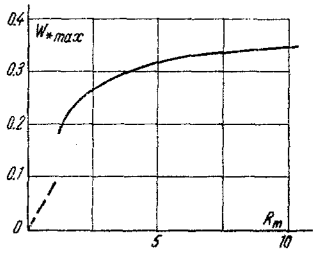

Fig. 10

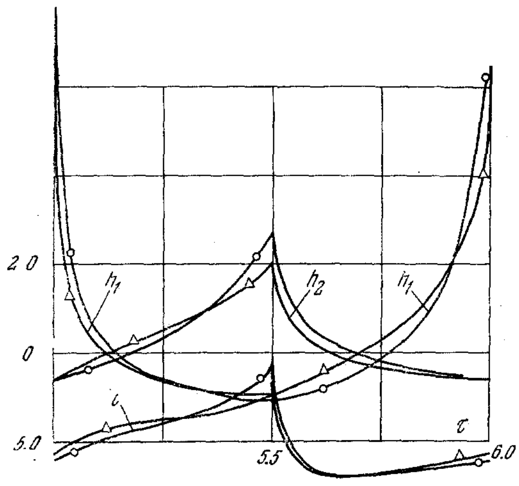

Fig. 11

electric energy goes into Joule heating. In the local sense, from (1.2), such a mode corresponds to the case of negative k, i.e., when **E** has the same direction as $(\mathbf{V} \times \mathbf{H})c^{-1}$.

For other values of β, the interval for θ is much smaller and includes only modes with positive energy liberation (Figs. 7, 9).

Figure 10 shows a graph of $2W_*/H^2_0 a$ as a function of R_m for the case $\lambda = 0, 8 = 1$ and maximum values of $2W_*/ /H^2_0 a$ obtained for four values of $\beta = 4, 2, 0.5$, and 0.2. This graph which is obtained for a given system assuming R_m is small when $\delta = 1$, is shown by the dashed line.

Calculations for $R_m \ll 1$(not given) show that the maximum energy W_*^{max} for fixed R_m occurs for a phase shift $\theta = \pi$; in this case, we see that $W_*^{max} > 0$ only if the periodic magnetic field has a constant component greater than the oscillation amplitude.

(b) Here, $\delta = 0$ for which the applied emf is zero and energy drain takes place to a winding with ohmic resistance. For numerical calculation the geometric parameters S_0, s_1, l and b_1 are the same as for case (a).

Since it is known that for $R_m \ll 1$ this variant is of no particular interest to the magnitude of usable energy, calculations were performed for $\beta = 2$ and $\beta = 0.5$ ($R_m = 2.4$ and 9.6) and different λ. Figure 11 shows the functions $h_1(\tau)$, $h_2(\tau)$, and $i(\tau)$ obtained after the system becomes periodic for two values of $\beta(R_m)$ and $\lambda = 5$ (the circles and trangles correspond respectively to $\beta = 0.5$ ($R_m = 9.6$) and $\beta = 2.0(R_m = 2.4)$).

For convenience, Fig. 8 and i, which show $2W_*/H^2_0\, a$ as a function of λ and $\eta(\lambda)$ for the case $\lambda = 0, \delta \neq 0$, also show $2W_*/H^2_0\, a$ as functions of θ and $\eta(\theta)$ for these values of R_m. It is clear that even for such large values of R_m (2.4 and 9.6) the liberated energy and the electric efficiency η are less than the same quantities when an external emf is used (for a purely ohmic load). For example, for $R_m = 2.4$, as is clear from Fig. 9, the maximum values of the electrical efficiency and the liberated electrical energy when an external emf is used are approximately two times greater than the maximum values of the same quantities when an ohmic load is applied. When employing such a comparison we should keep in mind the fact that when working with an external emf one optimization parameter remains unused (the values $\delta = 1$ used in calculations is, of course, not optium) whereas for an ohmic load all optimization possibilities are exhausted.

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